▶ PAUL SHAFER, *An introduction to reverse mathematics.*
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Reverse mathematics aims to determine precisely which axioms are required to prove which theorems. The name "reverse mathematics" comes from the idea that one shows a strong axiom A is necessary to prove a theorem T by showing that A can be deduced from T over a weak set of basic axioms. The proof of axiom A from theorem T is called a "reversal," which puts the "reverse" in "reverse mathematics." This course gives an overview of the recursion-theoretic approach to reverse mathematics in second-order arithmetic.

Part 1: *Proofs and reversals.* We use the classic Tietze extension theorem to introduce the main subsystems of second-order arithmetic, proofs in these systems, and reversals to these systems. The Tietze extension theorem also illustrates that the strength of a "theorem" can vary considerably depending on exactly how the theorem is phrased.

Part 2: *Models and separations.* Reverse mathematics is not only about equivalences. It is also about showing when one theorem T does *not* suffice to prove some other theorem S. To do this, one typically shows that there are objects that theorem T asserts to exist which do not compute any of the objects that theorem S asserts to exist, and then one iterates this idea to build a model of T that is not a model of S. We illustrate this process via Dzhafarov and Jockusch's proof of Seetapun's theorem separating Ramsey's theorem for pairs from arithmetical comprehension.

Part 3: *Induction* (time permitting). If time permits, we revisit proofs, reversals, models, and separations with an eye toward induction axioms and non-standard models. We discuss a theorem of Rival and Sands that turns out to be equivalent to Σ_2 induction plus the ascending/descending sequence principle. We also discuss principles concerning so-called "diagonally non-recursive functions" that are equivalent to weak Koenig's lemma in the presence of Σ_2 induction, but are weaker in the presence of only Σ_2 bounding.